Mixed Integer Programming for Modelling Fairness Constraints

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Introduction

- Morality of Machine Learning models in **real** datasets.
- *Support Vector Machine* (SVM) and Quadratic Programs (QP).
- Mixed Integer Programming (MIP) is NP-complete.
- Equal Opportunity constraints turn QP into MIP problem with quadratic constraints (QCQP).

Methods

The MIP optimization problem

- Extraction label $\alpha_i \in \{0,1\}$
- True label $y_i \in \{-1,+1\}$
- Protected class label $g_i \in \{0,1\}$ (i.e.,age, gender, race)
- Number of unprotected and protected individuals $\#N$ resp., $\#P$
- Indicator function $\mathbf{1}(u)$
- Sign function $\mathbf{sign}(x)$

$$\min_{w,b,\alpha,\zeta,z} \beta\sum_{i=1}^{n}\alpha_i + (1-\beta)\left(\frac{w^T w}{2} + \frac{C}{n}\sum_{i=1}^{n}\zeta_i\right) \tag{2}$$

subject to

$$\begin{aligned} &\Big|\frac{1}{\#P}\sum_{i\in\mathbb{P}}(1-\alpha_i)\mathbf{1}(sign(w^T x_i + b) = +1) - \\ &\quad \frac{1}{\#N}\sum_{i\in\mathbb{N}}(1-\alpha_i)\mathbf{1}(sign(w^T x_i + b) = +1)\Big| \leq \Delta \qquad \forall i = \overline{1,n} \end{aligned} \tag{3}$$

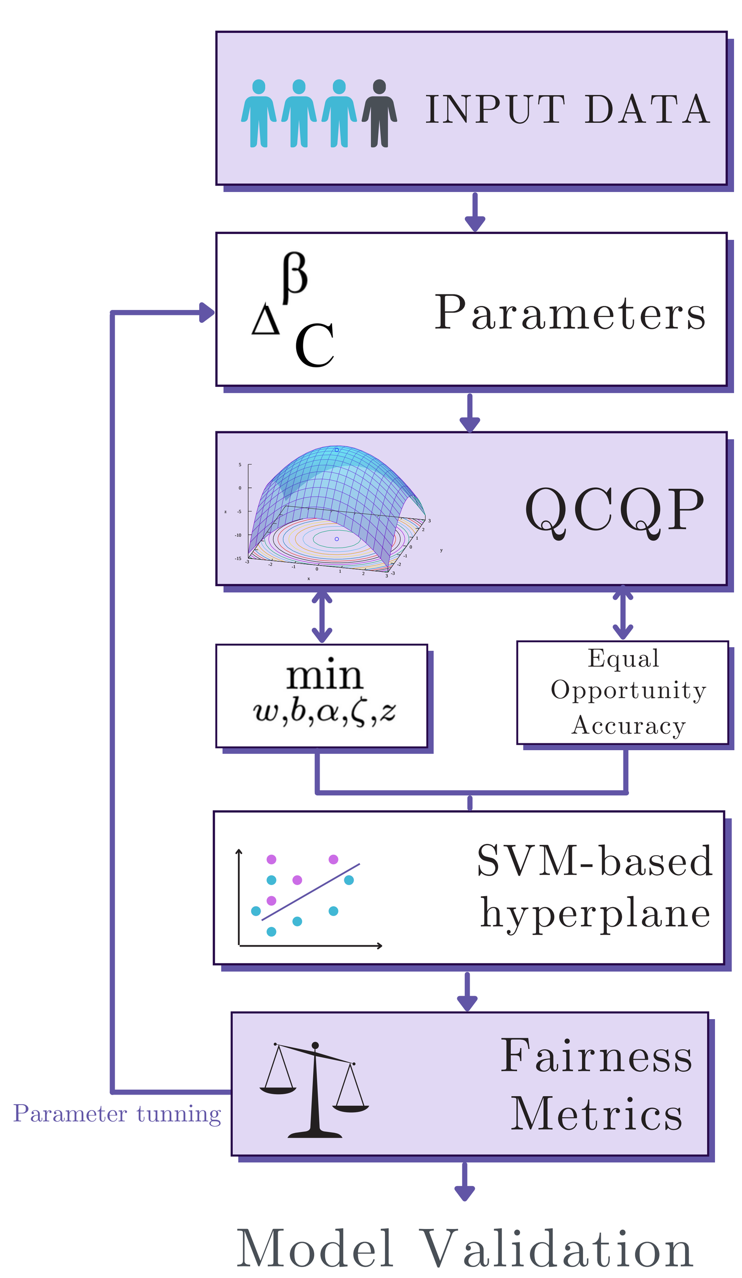
$$y_i(w^T x_i + b) \geq 1 - \zeta_i \qquad \forall i = \overline{1,n} \tag{4}$$

$$\alpha_i \in \{0,1\} \qquad \forall i = \overline{1,n} \tag{5}$$

$$w \in \mathbb{R}^d \tag{6}$$

$$b \in \mathbb{R} \tag{7}$$

Implementation and Results



	QCQP		Original SVM	
Training size	Accuracy	Equal Opportunity	Accuracy	Equal Opportunity
100	70.51	100.52	69.87	98.45
500	72.24	98.25	73.12	94.66

Table 1. German dataset.

	QCQP		Original SVM	
Training size	Accuracy	Equal Opportunity	Accuracy	Equal Opportunity
100	60.82	99.28	60.55	91.39
500	64.57	79.69	64.74	79.81

Table 2. COMPAS dataset.

Tables 1 and 2: QCQP-SVM metrics(left) vs. original SVM(right)
β = 0.5, time limit 10m. For size 100: Δ= 0.001, for size 500: Δ= 0.05.

Fairness

Here, the selected branch of Fairness is **Equal Opportunity**, i.e. *The probability of getting a positive outcome $\hat{y}_i$ is independent of protected class label g_i and conditional on the true label y_i being positive, for a relatively small deviation* $\Delta \in \mathbb{R}$.

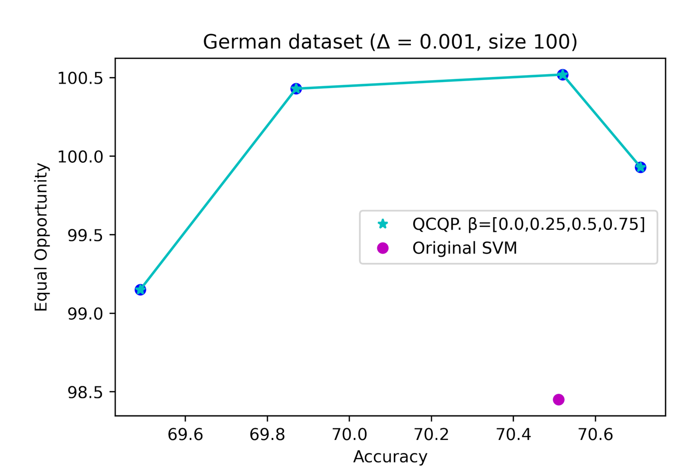
$$\begin{aligned} &|\mathbb{P}(\hat{y}_i = 1 | g_i = 0, y_i = 1) - \\ &\mathbb{P}(\hat{y}_i = 1 | g_i = 1, y_i = 1)| \leq \Delta, \\ &\forall i = 1, \dots, n \end{aligned}$$

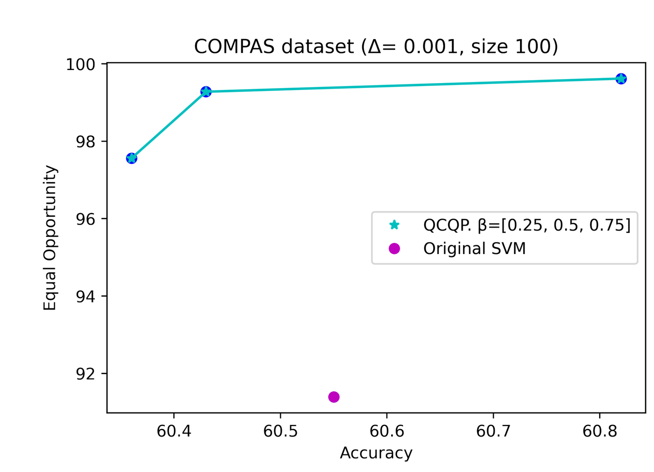
$g_i, \hat{y}_i, y_i \in \{0,1\}$
$\Delta \in \mathbb{R}^+$

Conclusions

· We propose a novel QCQP formulation to build an SVM-type classifier including fairness constraints.
· Our results show an improvement in fairness without important loss in accuracy.
· The trade-off detween time and training sample size is due to the constructed quadratic matrix. The trade between Accuracy and Equal Opportunity is eventually gentle.
· Generally, minimising extraction determines that no individual is extracted from the dataset, instead, hyperplane is skewed.
· QCQP problem can also be used to achive Equal Treatement.

Metrics visualization





References

[1] M. Olfat and A. Aswani. Spectral algorithms for computing fair support vector machines. PMLR, 2018.

[2] IBM ILOG Cplex. *International Business Machines Corporation*, 46(53):157, 2009.

Github Project

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