Mixed Integer Programming for Modelling Fairness Constraints

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Introduction

- Morality of Machine Learning models in real datasets.
- Support Vector Machine (SVM) and Quadratic Programs (QP).
- Mixed Integer Programming (MIP) is NP-complete.
- Equal Opportunity constraints turn QP into MIP problem with quadratic constraints (QCQP).

Methods

The MIP optimization problem

- Extraction label \(y_i \in \{0, 1\}\)
- True label \(y_i \in \{-1, +1\}\)
- Protected class label \(g_i \in \{0, 1\}\) (i.e., age, gender, race)
- Number of unprotected and protected individuals \(#N\) resp., \(#P\)
- Indicator function \(1(u)\)
- Sign function \(\text{sign}(x)\)

\[
\min_{w, b, \alpha, \xi} \beta \sum_{i=1}^{n} \alpha_i + (1 - \beta) \left( \frac{w^T w}{2} + C \sum_{i=1}^{n} \xi_i \right) \\
\text{subject to} \\
\frac{1}{#P} \sum_{i \in P} (1 - \alpha_i) 1(\text{sign}(w^T x_i + b) = +1) - \frac{1}{#N} \sum_{i \in N} (1 - \alpha_i) 1(\text{sign}(w^T x_i + b) = +1) \leq \Delta \\
\forall i = 1, \ldots, n \\
y_i(w^T x_i + b) \geq 1 - \xi_i \\
\alpha_i \in \{0, 1\} \\
w \in \mathbb{R}^d \\
b \in \mathbb{R} \\
(\Delta) 
\]

Implementation and Results

Fairness

Here, the selected branch of Fairness is Equal Opportunity, i.e. The probability of getting a positive outcome \(y_i\), is independent of protected class label \(g_i\), and conditional on the true label \(y_i\) being positive, for a relatively small deviation \(\Delta \in \mathbb{R}\).

\[
\mathbb{P}(y_i = 1|g_i = 0, y_i = 1) = \mathbb{P}(y_i = 1|g_i = 1, y_i = 1) \leq \Delta, \\
\forall i = 1, \ldots, n \\
g_i, y_i \in \{0, 1\} \\
\Delta \in \mathbb{R}^+ 
\]

Conclusions

- We propose a novel QCQP formulation to build an SVM-type classifier including fairness constraints.
- Our results show an improvement in fairness without important loss in accuracy.
- The trade-off between time and training sample size is due to the constructed quadratic matrix. The trade between Accuracy and Equal Opportunity is eventually gentle.
- Generally, minimising extraction determines that no individual is extracted from the dataset, instead, hyperplane is skewed.
- QCQP problem can also be used to achieve Equal Treatment.

Metrics visualization

References